Arithmetic Progression
First term is 1, difference is 5. Fifth term?

Arithmetic Progression
First term = 2, difference = 3. 8th term?

Arithmetic Progression
Sum of the first 7 multiples of 5.

Arithmetic Progression.
Sum of the first 6 natural numbers.

Arithmetic Progression.
Sum of the first 50 natural numbers.

Arithmetic Progression.
Sum of the first 50 even numbers.

Arithmetic Progression.
Sum of the first 5 natural numbers.

Arithmetic Progression.
Sum of the first 20 even numbers.

Arithmetic Progression.
Sum of the first 100 natural numbers.

Arithmetic Progression.
Sum of the first 10 multiples of 3.

$$23 = 2 + (7)3$$
$$t_n = t_1 + (n-1)d$$

$$21 = 1 + 4(5)$$
$$t_n = t_1 + (n-1)d$$

$$21 = \frac{6}{2}(1 + 6)$$
$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$140 = \frac{7}{2}(5 + 5 + (6)5)$$
$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(t_1 + t_1 + (n-1)d)$$

$$2550 = \frac{50}{2}(2 + 2 + (49)(2)$$
$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(t_1 + t_1 + (n-1)d)$$

$$1275 = \frac{50}{2}(1 + 50)$$
$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$420 = \frac{20}{2}(2 + 2 + (19)2)$$
$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(t_1 + t_1 + (n-1)d)$$

$$15 = \frac{5}{2}(1 + 5)$$
$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$165 = \frac{10}{2}(3 + 3 + (9)3)$$
$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(t_1 + t_1 + (n-1)d)$$

$$5050 = \frac{100}{2}(1 + 100)$$
$$S_n = \frac{n}{2}(t_1 + t_n)$$

Arithmetic Progression.
First term is 8, difference is 5. 4th term?

Arithmetic Progression.
First term = 3, difference = 4. 11th term?

Arithmetic Progression.
First term = 2, difference = 7. 3rd term?

Arithmetic Progression.
First term = 100, difference = 6. 7th term?

Geometric Progression First term = 1.
Common ratio = 1.5
5th term?

Geometric Progression First term = 2.
Common ratio = 2
6th term?

Geometric Progression First term = 1.
Common ratio = 1.10
8th term?

Geometric Progression First term = 1000.
Common ratio = 1.03
20th term?

Geometric Progression First term = 5000.
Common ratio = 1.07
30 term?

Geometric Progression First term = 1.
Common ratio = 1.0525
15th term?

$$43 = 3 + (10)4$$
$$t_n = t_1 + (n-1)d$$

$$23 = 8 + (3)5$$
$$t_n = t_1 + (n-1)d$$

$$136 = 100 + 6(6)$$
$$t_n = t_1 + (n-1)d$$

$$16 = 2 + (2)7$$
$$t_n = t_1 + (n-1)d$$

$$t_n = t_1 r^{n-1} = 2(2^5) = 64$$

$$t_n = t_1 r^{n-1} = 1(1.5^4) = 38.44$$

$$t_n = t_1 r^{n-1} = 1000(1.03^{19}) = 1806.11$$

$$t_n = t_1 r^{n-1} = 1(1.1^7) = 1.949$$

$$t_n = t_1 r^{n-1} = 1(1.0525^{14}) = 2.046$$

$$t_n = t_1 r^{n-1} = 5000(1.07^{29}) = 38,061.$$

Geometric Progression, first term $= t_1$, common ratio $= r$. Sum of first n terms?

Sum of first n terms of geometric progression with first term $= t_1$

$$S_n = t_1 \frac{???}{r-1} \text{ or } t_1 \frac{1-r^n}{1-r}$$

Sum of first n terms of geometric progression with first term $= t_1$

$$S_n = t_1 \frac{r^n-1}{???} \text{ or } t_1 \frac{1-r^n}{1-r}$$

Sum of first n terms of geometric progression with first term $= t_1$

$$S_n = ??? \frac{r^n-1}{r-1} \text{ or } t_1 \frac{1-r^n}{1-r}$$

Sum of first n terms of geometric progression with first term $= t_1$

$$S_n = t_1 \frac{r^n-1}{r-1} \text{ or } ??? \frac{1-r^n}{1-r}$$

Sum of first n terms of geometric progression with first term $= t_1$

$$S_n = t_1 \frac{r^n-1}{r-1} \text{ or } t_1 \frac{???}{1-r}$$

Sum of first n terms of geometric progression with first term $= t_1$

$$S_n = t_1 \frac{r^n-1}{r-1} \text{ or } t_1 \frac{1-r^n}{???}$$

$$a(0) = ???$$

$$a(1) = ???$$

$$A(t) = ??? a(t)$$

$r^n - 1$	$t_1 \dfrac{r^n - 1}{r - 1}$ or $t_1 \dfrac{1 - r^n}{1 - r}$
t_1	$r - 1$
$1 - r^n$	t_1
1	$1 - r$
k, a constant	$1 + i$

v is the ??? of $1+i$

Reciprocal of $1+i$

Reciprocal of v

The amount of money that one unit invested at the beginning of a period will earn during the period, with interest being paid at the end of the period.

The ??? for a period is the amount of interest earned in one period divided by the principal at the beginning of the period.

The effective rate of interest for a period is ??? divided by the principal at the beginning of the period.

The effective rate of interest for a period is the amount of interest earned in one period divided by ???.

The ??? is the ratio of the amount of interest earned during the period to the amount invested at the end of the period.

The effective rate of discount is the ratio of ??? to the amount invested at the end of the period.

The effective rate of discount is the ratio of the amount of interest earned during the period to ???.

v

reciprocal

Effective Rate of Interest=*i*

$1 + i$

the amount of interest earned in one period
$i_n = \frac{A(n)-A(n-1)}{A(n-1)}$

effective rate of interest
$i_n = \frac{A(n)-A(n-1)}{A(n-1)}$

effective rate of discount = *d*
$i_n = \frac{A(n)-A(n-1)}{A(n-1)}$

the principal at the beginning of the period
$i_n = \frac{A(n)-A(n-1)}{A(n-1)}$

the amount invested at the end of the period
$d_n = \frac{A(n)-A(n-1)}{A(n)}$

the amount of interest earned during the period
$d_n = \frac{A(n)-A(n-1)}{A(n)}$

$$??? = \frac{A(n) - A(n-1)}{A(n-1)}$$

Financial Mathematics Interest Theory

$$i_n = \frac{??? - A(n-1)}{A(n-1)}$$

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Financial Mathematics Interest Theory

$$i_n = \frac{A(n) - ???}{A(n-1)}$$

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Financial Mathematics Interest Theory

$$i_n = \frac{A(n) - A(n-1)}{???}$$

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Financial Mathematics Interest Theory

$$??? = \frac{A(n) - A(n-1)}{A(n)}$$

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Financial Mathematics Interest Theory

$$d_n = \frac{??? - A(n-1)}{A(n)}$$

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Financial Mathematics Interest Theory

$$d_n = \frac{A(n) - ???}{A(n)}$$

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Financial Mathematics Interest Theory

$$d_n = \frac{A(n) - A(n-1)}{???}$$

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Financial Mathematics Interest Theory

??? =
(Balance at start of year)(Interest Rate)

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Financial Mathematics Interest Theory

Compound interest earned during a year =
(???)(Interest Rate)

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$A(n)$

i_n

$A(n-1)$

$A(n-1)$

$A(n)$

d_n

$A(n)$

$A(n-1)$

Balance at start of year
For year *m* this is
$i(1+i)^{(m-1)}$

Compound interest earned during a year
For year *m* this is
$i(1+i)^{(m-1)}$

Financial Mathematics Interest Theory

Compound interest earned during a year = (Balance at start of year)(???)

Financial Mathematics Interest Theory

\$100 is invested at $i = 4\%$ today. Value in 10 years?

Financial Mathematics Interest Theory

\$50 is invested at $i = 3.25\%$ today. Value in 25 years?

Financial Mathematics Interest Theory

\$1 is invested at $i = 7.10\%$ today. Value in 40 years?

Financial Mathematics Interest Theory

\$30 is invested at $i = 1.5\%$ today. Value in 10 years?

Financial Mathematics Interest Theory

\$10500 is invested at $i = 4.75\%$ today. Value in 30 years?

Financial Mathematics Interest Theory

\$100,000 is invested at $i = 5\%$ today. Value in 10 years?

Financial Mathematics Interest Theory

With compound interest $i = 0.05$, what is present value of 10,000 dollars in 40 years?

Financial Mathematics Interest Theory

With compound interest $i = 0.04$, what is present value of 20,000 dollars in 20 years?

Financial Mathematics Interest Theory

With compound interest $i = 0.03$, what is present value of 2,000 dollars in 9 years?

$$100(1.04)^{10} = 148.02$$
$$a(t) = (1+i)^t$$
$$FV = PV(1+i)^t$$

Interest Rate
For year *m* this is
$i(1+i)^{(m-1)}$

$$1(1.071)^{40} = 15.54$$
$$a(t) = (1+i)^t$$
$$FV = PV(1+i)^t$$

$$50(1.0325)^{25} = 111.23$$
$$a(t) = (1+i)^t$$
$$FV = PV(1+i)^t$$

$$10500(1.0475)^{30} = 42248.40$$
$$a(t) = (1+i)^t$$
$$FV = PV(1+i)^t$$

$$30(1.015)^{10} = 34.82$$
$$a(t) = (1+i)^t$$
$$FV = PV(1+i)^t$$

$$\frac{10{,}000}{1.05^{40}}$$
$$PV = \frac{FV}{(1+i)^n}$$

$$100,000(1.05)^{10} = 162,889.46$$
$$a(t) = (1+i)^t$$
$$FV = PV(1+i)^t$$

$$\frac{2{,}000}{1.03^{9}}$$
$$PV = \frac{FV}{(1+i)^n}$$

$$\frac{20{,}000}{1.04^{20}}$$
$$PV = \frac{FV}{(1+i)^n}$$

With compound interest $i = 0.02$, what is present value of 1,000 dollars in 10 years?

With compound interest i = 0.01, what is present value of 10,000 dollars in 5 years?

$$v = 0.99$$
$$d = ???$$

$$v = 0.98$$
$$d = ???$$

$$v = 0.97$$
$$d = ???$$

$$v = 0.92$$
$$d = ???$$

$$v = 0.85$$
$$d = ???$$

$$??? - d = id$$

$$i - ??? = id$$

$$i - d = ???d$$

$\frac{10{,}000}{1.01^5}$

$PV = \frac{FV}{(1+i)^n}$

$\frac{1{,}000}{1.02^{10}}$

$PV = \frac{FV}{(1+i)^n}$

0.02

$d = 1 - v$

0.01

$d = 1 - v$

0.08

$d = 1 - v$

0.03

$d = 1 - v$

i

0.15

$d = 1 - v$

i

d

$$i - d = i???$$

$$d < d^{(m)} < \delta < i^{(m)} < ???$$

$$??? < d^{(m)} < \delta < i^{(m)} < i$$

$$d < ??? < \delta < i^{(m)} < i$$

$$d < d^{(m)} < ??? < i^{(m)} < i$$

$$d < d^{(m)} < \delta < ??? < i$$

$$d = 0.10$$
$$i = ???$$

$$d = 0.08$$
$$i = ???$$

$$d = 0.06$$
$$i = ???$$

$$d = 0.04$$
$$i = ???$$

i	d
$d^{(m)}$	d
$i^{(m)}$	δ
$i = \frac{d}{(1-d)} = 0.0870$	$i = \frac{d}{(1-d)} = 0.1111$
$i = \frac{d}{(1-d)} = 0.0417$	$i = \frac{d}{(1-d)} = 0.0638$

$$d = 0.02$$
$$i = ???$$

$$d = 0.01$$
$$i = ???$$

The present value of $50000 payable in 5 years at an effective annual discount rate of 2%.

The present value of $50000 payable in 30 years at an effective annual discount rate of 5%.

The present value of $10000 payable in 20 years at an effective annual discount rate of 3%

The present value of $1000 payable in 15 years at an effective annual discount rate of 4%.

The present value of $1000 payable in 10 years at an effective annual discount rate of 6%.

The present value of $1 payable in 10 years at an effective annual discount rate of 12%.

Rate is 2% per quarter.
Annual Nominal rate = $i^{(4)}$ = ???

Rate is 1% per month.
Effective rate i = ???

$$i = \frac{d}{(1-d)} = 0.0101$$

$$i = \frac{d}{(1-d)} = 0.0204$$

$$50000(1-0.05)^{30}$$
$$PV = FVv^n$$

$$50000(1-0.02)^{5}$$
$$PV = FVv^n$$

$$1000(1-0.04)^{15}$$
$$PV = FVv^n$$

$$10000(1-0.03)^{20}$$
$$PV = FVv^n$$

$$(1-0.12)^{10}$$
$$PV = FVv^n$$

$$1000(1-0.06)^{10}$$
$$PV = FVv^n$$

$$(1.01)^{12} - 1 = 0.1268$$
$$= (1 + \frac{i^{(m)}}{m})^m - 1$$

8%

Rate is 1% per month.
Annual Nominal rate = $i^{(12)}$ = ???

Rate is 0.5% per month.
Effective rate i = ???

Rate is 0.5% per month.
Annual Nominal rate = $i^{(12)}$ = ???

Rate is 0.25% per month.
Effective rate i = ???

Rate is 0.25% per month.
Annual Nominal rate = $i^{(12)}$ = ???

Nominal Annual rate is $i^{(12)} = 6\%$.
Monthly rate =???

Nominal Annual rate is $i^{(4)} = 8\%$.
Quarterly rate =???

Nominal Annual rate is $i^{(2)} = 6\%$.
Semi annual rate =???

Nominal Annual rate is $i^{(2)} = 2\%$.
Semi annual rate =???

Nominal Annual rate is $i^{(12)} = 12\%$.
Monthly rate =???

$1.005^{12} = 6.17\%$	12%
$(1.0025)^{12} - 1 = 0.0304$ $= (1 + \frac{i^{(m)}}{m})^m - 1$	6%
0.5%	3%
3%	2%
1%	1%

Nominal Annual rate is $i^{(12)} = 6\%$.
Effective rate $i = ???$

Nominal Annual rate is $i^{(4)} = 8\%$.
Effective rate $i = ???$

Nominal Annual rate is $i^{(2)} = 6\%$.
Effective rate $i = ???$

Nominal Annual rate is $i^{(2)} = 2\%$.
Effective rate $i = ???$

Nominal Annual Rate is 6%, convertible 12 times per year.
Effective rate $i = ???$

Amount of interest earned in year 8 for $50 deposited at $i = 15\%$.

Amount of interest earned in year 6 for $10000 deposited at 6%.

Amount of interest earned in year 30 for $500 deposited at 4.5%.

Amount of interest earned in year 3 for $100000 deposited at $i = 1.5\%$.

Amount of interest earned in year 25 for $50000 deposited at $i = 8\%$.

$$(1+\frac{0.08}{4})^4-1=0.0824$$
$$(1+\frac{i^{(m)}}{m})^m-1$$

$$(1+\frac{0.06}{12})^{12}-1=0.0617$$
$$(1+\frac{i^{(m)}}{m})^m-1$$

$$(1+\frac{0.02}{2})^2-1=0.0201$$
$$(1+\frac{i^{(m)}}{m})^m-1$$

$$(1+\frac{0.06}{2})^2-1=0.0609$$
$$(1+\frac{i^{(m)}}{m})^m-1$$

$$(50)(0.15)(1.15)^7$$
$$i(1+i)^{(m-1)}$$
=rate of interest * total amount at beginning of year

$$(1+\frac{0.06}{12})^{12}-1=0.061$$
$$=(1+\frac{i^{(m)}}{m})^m-1$$

$$(500)(0.045)(1.045)^{29}$$
$$i(1+i)^{(m-1)}$$
=rate of interest * total amount at beginning of year

$$(10000)(0.06)(1.06)^5$$
$$i(1+i)^{(m-1)}$$
=rate of interest * total amount at beginning of year

$$(50000)(0.08)(1.08)^{24}$$
$$i(1+i)^{(m-1)}$$
=rate of interest * total amount at beginning of year

$$(100000)(0.015)(1.015)^2$$
$$i(1+i)^{(m-1)}$$
=rate of interest * total amount at beginning of year

Amount of interest earned in year 15 for \$3000 deposited at $i = 2\%$.

\$8 today yields \$15 in 18 years.
Interest Rate?

\$5 today yields \$8 in 12 years.
Interest Rate?

\$10000 today yields \$100000 in 40 years.
Interest Rate?

\$1000 today yields \$5000 in 50 years.
Interest Rate?

\$1000 today yields \$5000 in 30 years.
Interest Rate?

\$1000 today yields \$2300 in 10 years.
Interest Rate?

\$1000 today yields \$1200 in 5 years.
Interest Rate?

\$10 today yields \$20 in 7 years.
Interest Rate?

\$1 today yields \$3 in 15 years
Interest Rate?

3.6%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

$$(3000)(0.02)(1.02)^{14} = 79.17$$

$$i(1+i)^{(m-1)}$$

=rate of interest * total amount at beginning of year

5.9%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

4%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

5.51%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

3.27%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

3.7%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

8.7%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

7.6%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

10.4%

$$i = (\frac{FV}{PV})^{\frac{1}{n}} - 1$$

$1 today yields $10 in 30 years.
Interest Rate?

$1 today yields $1.50 in 5 years.
Interest Rate?

$\bullet$	i	v	d	δ
i	i	???	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	???	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	???
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	???	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	???	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	???
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	???	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	???	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

8.4%

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

8%

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

$$i = \frac{d}{1-d}$$

$$i = \frac{1}{v} - 1$$

$$v = (1+i)^{-1}$$

$$i = e^{\delta} - 1$$

$$v = e^{-\delta}$$

$$v = 1 - d$$

$$d = 1 - v$$

$$d = \frac{i}{1+i}$$

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	???
δ	$ln(1+i)$	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	???	$-lnv$	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	???	$-ln(1-d)$	δ

$\bullet$	i	v	d	δ
i	i	$\frac{1}{v}-1$	$\frac{d}{1-d}$	$e^{\delta}-1$
v	$(1+i)^{-1}$	v	$1-d$	$e^{-\delta}$
d	$\frac{i}{1+i}$	$1-v$	d	$1-e^{-\delta}$
δ	$ln(1+i)$	$-lnv$	???	δ

With compound interest, value at $t = 1$ is \$1. Value at $t = 0$?

With compound discount, value at $t = 1$ is \$1. Value at $t = 0$?

With force of interest, value at $t = 1$ is \$1. Value at $t = 0$?

With nominal interest, value at $t = 1$ is \$1. Value at $t = 0$?

With nominal discount, value at $t = 1$ is \$1. Value at $t = 0$?

With compound interest, value at $t = 0$ is \$1. Value at $t = 1$?

$\delta = ln(1+i)$

$d = 1 - e^{-\delta}$

$\delta = -ln(1-d)$

$\delta = -lnv$

$1 - d$

$(1+i)^{-1}$, or v

$(1 + \frac{i^m}{m})^{-m}$

$e^{-\delta}$

$1 + i$

$(1 - \frac{d^{(m)}}{m})^m$

With compound discount, value at $t = 0$ is \$1. Value at $t = 1$?

With force of interest, value at $t = 0$ is \$1. Value at $t = 1$?

With nominal interest, value at $t = 0$ is \$1. Value at $t = 1$?

With nominal discount, value at $t = 0$ is \$1. Value at $t = 1$?

time	$t = 0$	$t = 1$
interest	???	\$1
discount	$1 - d$	\$1
force of interest	$e^{-\delta}$	\$1
nominal interest	$(1 + \frac{i^m}{m})^{-m}$	\$1
nominal discount	$(1 - \frac{d^{(m)}}{m})^m$	\$1

time	$t = 0$	$t = 1$
interest	$(1 + i)^{-1}$	\$1
discount	???	\$1
force of interest	$e^{-\delta}$	\$1
nominal interest	$(1 + \frac{i^m}{m})^{-m}$	\$1
nominal discount	$(1 - \frac{d^{(m)}}{m})^m$	\$1

time	$t = 0$	$t = 1$
interest	$(1 + i)^{-1}$	\$1
discount	$1 - d$	\$1
force of interest	???	\$1
nominal interest	$(1 + \frac{i^m}{m})^{-m}$	\$1
nominal discount	$(1 - \frac{d^{(m)}}{m})^m$	\$1

time	$t = 0$	$t = 1$
interest	$(1 + i)^{-1}$	\$1
discount	$1 - d$	\$1
force of interest	$e^{-\delta}$	\$1
nominal interest	???	\$1
nominal discount	$(1 - \frac{d^{(m)}}{m})^m$	\$1

time	$t = 0$	$t = 1$
interest	$(1 + i)^{-1}$	\$1
discount	$1 - d$	\$1
force of interest	$e^{-\delta}$	\$1
nominal interest	$(1 + \frac{i^m}{m})^{-m}$	\$1
nominal discount	???	\$1

time	$t = 0$	$t = 1$
interest	\$1	???
discount	\$1	$\frac{1}{1-d}$
force of interest	\$1	e^{δ}
nominal interest	\$1	$(1 + \frac{i^m}{m})^m$
nominal discount	\$1	$(1 - \frac{d^{(m)}}{m})^{-m}$

$$e^{\delta}$$

$$(1-d)^{-1}$$

$$(1-\frac{d^{(m)}}{m})^{-m}$$

$$(1+\frac{i^{m}}{m})^{m}$$

$$1-d$$

$$(1+i)^{-1}$$

$$(1+\tfrac{i^{m}}{m})^{-m}$$

$$e^{-\delta}$$

$$(1+i)$$

$$(1-\tfrac{d^{(m)}}{m})^{m}$$

time	$t=0$	$t=1$
interest	\$1	$(1+i)$
discount	\$1	???
force of interest	\$1	e^{δ}
nominal interest	\$1	$(1+\frac{i^m}{m})^m$
nominal discount	\$1	$(1-\frac{d^{(m)}}{m})^{-m}$

time	$t=0$	$t=1$
interest	\$1	$(1+i)$
discount	\$1	$\frac{1}{1-d}$
force of interest	\$1	???
nominal interest	\$1	$(1+\frac{i^m}{m})^m$
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force of interest	\$1	e^{δ}
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discount	\$1	$\frac{1}{1-d}$
force of interest	\$1	e^{δ}
nominal interest	\$1	$(1+\frac{i^m}{m})^m$
nominal discount	\$1	???

$$???^t = a^{-1}(t) = (1-d)^t = (1+i)^{-t} = \left(\frac{1}{1+i}\right)^t$$

$$v^t = ???^{-1}(t) = (1-d)^t = (1+i)^{-t} = \left(\frac{1}{1+i}\right)^t$$

$$v^t = a^{-1}(t) = (1-???)^t = (1+i)^{-t} = \left(\frac{1}{1+i}\right)^t$$

$$v^t = a^{-1}(t) = (1-d)^t = (???)^{-t} = \left(\frac{1}{1+i}\right)^t$$

$$v^t = a^{-1}(t) = (1-d)^t = (1+i)^{-t} = \left(\frac{1}{???}\right)^t$$

$$\left(1+\frac{i^{(m)}}{m}\right)^{???} = \left(1-\frac{d^{(p)}}{p}\right)^{-p} = 1+i$$

e^{δ}	$\frac{1}{1-d}$
$(1-\frac{d^{(m)}}{m})^{-m}$	$(1+\frac{i^{m}}{m})^{m}$
a	v
$1+i$	d
m	$1+i$

$$\left(1+\frac{i^{(m)}}{m}\right)^m = \left(1-\frac{d^{(p)}}{p}\right)^{???} = 1+i$$

$$\left(1+\frac{i^{(m)}}{m}\right)^m = \left(1-\frac{d^{(p)}}{p}\right)^{-p} = ???$$

When m = p, then:
$$\left(1+\frac{i^{(m)}}{m}\right) = \left(1-\frac{d^{(p)}}{p}\right)^{???} =$$

Force of ??? $= \delta_t = \frac{a'(t)}{a(t)} = \frac{A'(t)}{A(t)}$

Force of Interest $= ??? = \frac{a'(t)}{a(t)} = \frac{A'(t)}{A(t)}$

Force of Interest $= \delta_t = \frac{???}{a(t)} = \frac{A'(t)}{A(t)}$

Force of Interest $= \delta_t = \frac{a'(t)}{???} = \frac{A'(t)}{A(t)}$

Force of Interest $= \delta_t = \frac{a'(t)}{a(t)} = \frac{???}{A(t)}$

Force of Interest $= \delta_t = \frac{a'(t)}{a(t)} = \frac{A'(t)}{???}$

Real Rate of Interest (i') with Inflation (r)
$$i' = \frac{i-???}{1+r}$$

1+i	-p
Interest	-1
$a'(t)$	δ_t
$A'(t)$	$a(t)$
r	$A(t)$

Real Rate of Interest (i') with Inflation (r)

$$i' = \frac{i-r}{1+???}$$

Real Rate of Interest (i') with Inflation (r)

$$i' = \frac{???}{1+r}$$

Real Rate of Interest (i') with Inflation (r)

$$i' = \frac{i-r}{???}$$

Real Rate of Interest (i') with Inflation (r)

$$i' = \frac{???-r}{1+r}$$

$i = 4\%$. Rate of Inflation $(r) = 1.5\%$

Real rate of Interest (i')?

$i = 6\%$. Rate of Inflation $(r) = 2.5\%$

Real rate of Interest (i')?

$i = 2\%$. Rate of Inflation $(r) = 0.5\%$

Real rate of Interest (i')?

$i = 12\%$. Rate of Inflation $(r) = 10.5\%$

Real rate of Interest (i')?

$i = 4.5\%$. Rate of Inflation $(r) = 4.25\%$

Real rate of Interest (i')?

$i = 1\%$. Rate of Inflation $(r) = 1.5\%$

Real rate of Interest (i')?

$i - r$

r

i

$1 + r$

$\frac{0.06-0.025}{1.025} = 3.41\%$
$i' = \frac{i-r}{1+r}$

$\frac{0.04-0.015}{1.015} = 2.46\%$
$i' = \frac{i-r}{1+r}$

$\frac{0.12-0.105}{1.105} = 1.36\%$
$i' = \frac{i-r}{1+r}$

$\frac{0.02-0.005}{1.005} = 1.49\%$
$i' = \frac{i-r}{1+r}$

$\frac{0.01-0.015}{1.015} = -0.49\%$
$i' = \frac{i-r}{1+r}$

$\frac{0.045-0.0425}{1.0425} = 0.24\%$
$i' = \frac{i-r}{1+r}$

$i = 2\%$. Rate of Inflation $(r) = 0.5\%$
Real rate of Interest (i')?

Financial Mathematics Interest Theory

$$\lim_{m\to\infty} i^{(m)}$$

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Financial Mathematics Interest Theory

$$\delta$$

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Financial Mathematics Interest Theory

$$??? = \frac{A'(t)}{A(t)} = \frac{a'(t)}{a(t)} = \frac{d}{dt}\ln A(t) = \frac{d}{dt}\ln a(t)$$

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Financial Mathematics Interest Theory

$$\delta_t = ??? = \frac{a'(t)}{a(t)} = \frac{d}{dt}\ln A(t) = \frac{d}{dt}\ln a(t)$$

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Financial Mathematics Interest Theory

$$\delta_t = \frac{A'(t)}{A(t)} = ??? = \frac{d}{dt}\ln A(t) = \frac{d}{dt}\ln a(t)$$

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Financial Mathematics Interest Theory

$$\delta_t = \frac{A'(t)}{A(t)} = \frac{a'(t)}{a(t)} = ???\ln A(t) = \frac{d}{dt}\ln a(t)$$

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Financial Mathematics Interest Theory

$$\delta_t = \frac{A'(t)}{A(t)} = \frac{a'(t)}{a(t)} = \frac{d}{dt}\ln A(t) = ???\ln a(t)$$

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Financial Mathematics Interest Theory

$$\delta_t = \frac{A'(t)}{A(t)} = \frac{a'(t)}{a(t)} = \frac{d}{dt}\ln A(t) = \frac{d}{dt}???$$

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Financial Mathematics Interest Theory

$$\frac{a'(t)}{a(t)}$$

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δ

$$\frac{0.02 - 0.005}{1.005} = 1.49\%$$

$$i' = \frac{i - r}{1 + r}$$

δ_t

Force of Interest, the measure of continuous interest at an instant of time.

$$\frac{a'(t)}{a(t)}$$

$$\frac{A'(t)}{A(t)}$$

$$\frac{d}{dt}$$

$$\frac{d}{dt}$$

δ_t

$\ln a(t)$

$$\delta_t = \frac{d}{dt}???$$

$$\frac{a'(t)}{a(t)} = \frac{d}{dt}???$$

$$??? = e^{\int_0^t \delta(s)ds}$$

$$a(t) = ???^{\int_0^t \delta(s)ds}$$

$$a(t) = e^{\int_0^{???} \delta(s)ds}$$

The accumulated value of $10,000 invested for 6 years with $\delta = 3\%$

The accumulated value of $5,000 invested for 12 years with $\delta = 2\%$

The accumulated value of $50,000 invested for 8 years with $\delta = 6\%$

The accumulated value of $10,000 invested for 30 years with $\delta = 4\%$

The accumulated value of $10 invested for 50 years with $\delta = 7\%$

$\ln a(t)$

$\ln a(t)$ or $\ln A(t)$

e

$a(t)$

$10,000e^{(6)(.03)} = 11,972.17$

t

$50,000e^{(8)(1.06)} = 80,803.72$

$5,000e^{(12)(.02)} = 6356.25$

$10e^{(50)(.07)} = 331.15$

$10,000e^{(30)(.04)} = 33,201.17$

$8 today yields $15 in 18 years.
Force of Interest?

$10 today yields $30 in 20 years.
Force of Interest?

$10000 today yields $11000 in 3 years.
Force of Interest?

$1 today yields $10 in 40 years.
Force of Interest?

$100 today yields $150 in 10 years.
Force of Interest?

$50 today yields $60 in 5 years.
Force of Interest?

In how many years will $5 grow to $20 with δ =5%?

In how many years will $10 grow to $20 with δ =12%?

In how many years will $100 grow to $110 with δ =3.25%?

In how many years will $1 grow to $5 with δ =8%?

$$10e^{20\delta} = 30$$

$$\delta = 5.49\%$$

$$8e^{18\delta} = 15$$

$$\delta = 3.49\%$$

$$1e^{40\delta} = 10$$

$$\delta = 5.76\%$$

$$10000e^{3\delta} = 11000$$

$$\delta = 3.18\%$$

$$50e^{5\delta} = 60$$

$$\delta = 3.6\%$$

$$100e^{10\delta} = 150$$

$$\delta = 4.1\%$$

$$10e^{0.12t} = 20$$

$$t = 5.78 \text{ years}$$

$$5e^{0.05t} = 20$$

$$t = 27.73 \text{ years}$$

$$1e^{0.05t} = 5$$

$$t = 20.11 \text{ years}$$

$$100e^{0.0325t} = 110$$

$$t = 2.93 \text{ years}$$

$$a(t) = e^{\int_0^t ???}$$

$$e^{\ln x}$$

$$\int \frac{1}{u} du$$

$$\int \frac{1}{(1+u)} du$$

$$\int \frac{1}{(1-u)} du$$

$$\int \frac{1}{(1+u)^{-2}} du$$

$$\int \frac{1}{(4+u)} du$$

$$\int \frac{1}{(1+2t)} dt$$

$$\int \frac{2}{(1+2t)} dt$$

$$\int \frac{2}{(3+5t)} dt$$

$$x$$

$$\delta(s)ds$$

$$\ln(1+u)+c$$

$$\ln(u)+c$$

$$-(1+u)^{-1}$$

$$-\ln(1-u)+c$$

$$\frac{1}{2}\ln(1+2t)+c$$

let u $=1+2t$. Then $du=2dt$. Then dt $=\frac{1}{2}du$. Hence we have $\frac{1}{2}\int\frac{1}{u}du=\frac{1}{2}\ln u+c$

$$\ln(4+u)+c$$

$$\frac{2}{5}\ln(3+5t)+c$$

let u $=3+5t$. Then $du=5dt$. Then dt $=\frac{1}{5}du$. Hence we have $\frac{2}{5}\int\frac{1}{u}du=\frac{2}{5}\ln u+c$

$$\ln(1+2t)+c$$

let u $=1+2t$. Then $du=2dt$. Then dt $=\frac{1}{2}du$. Hence we have $\frac{2}{2}\int\frac{1}{u}du=\ln u+c$

$$\int \frac{3}{(2+3t)}dt$$

$$\delta_t = 0.06t^2$$
$$a(t) = ???$$

$$\delta_t = 0.01t$$
$$a(t) = ???$$

$$\delta_t = \frac{1}{1+t}$$
$$a(t) = ???$$

$$\delta_t = 0.1t$$
$$a(t) = ???$$

$$\delta_t = \frac{t^2}{30}$$
$$a(t) = ???$$

$$e^{0.02t^3}$$

$$e^{\int_0^t 0.06s^2 ds} = e^{[0.02s^3]_0^t}$$

$$\ln(2+3t)+c$$

let u $= 2+3t$. Then $du = 3dt$. Then dt $= \frac{1}{3}du$. Hence we have $\frac{3}{3}\int \frac{1}{u}du = \ln u + c$

$$1+t$$

$$e^{\int_0^t \frac{1}{1+s} ds} = e^{[ln(1+s)]_0^t}$$

$$e^{0.005t^2}$$

$$e^{\int_0^t 0.01s ds} = e^{[0.005s^2]_0^t}$$

$$e^{0.0111t^3}$$

$$e^{\int_0^t \frac{1}{30}s^2 ds} = e^{[\frac{1}{90}s^3]_0^t}$$

$$e^{0.05t^2}$$

$$e^{\int_0^t 0.1s ds} = e^{[0.05s^2]_0^t}$$